3.12 Double-Angle Identities

Double-angle Identities:

$$\sin 2u = 2\sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 2\cos^2 u - 1$$

$$\cos 2u = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

Ex.

Proving a Double-Angle Identity:

Prove the identity: $\sin 2u = 2\sin u \cos u$

$$\sin 2u = \sin(u+u)$$

$$= \sin u \cos u + \cos u \sin u = 2\sin u \cos u$$

Proving an Identity

Prove the identity: $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$$\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$

$$= 1(\cos^2\theta - \sin^2\theta) = \cos 2\theta$$

Using a Double-Angle Identity

Find all solutions to the equation in the interval $[0,2\pi)$.

a) $\sin 2x = \sin x$

$$\sin 2x - \sin x = 0$$

$$2\sin x \cos x - \sin x = 0$$

$$\sin x(2\cos x - 1) = 0$$

So $\sin x = 0$ or $\cos x = \frac{1}{2}$ when $x = 0, \pi, \frac{\pi}{3}, or \frac{5\pi}{3}$.